Using the peculiar properties of quantum entanglement to teleport quantum states

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Introduction

"In the beginning there were only probabilities."

– Martin Rees

What to think when you see the following math

This is a **ket** (it's a vector)

$$|\phi\rangle$$

It represents a quantum state.

$$*$$

What to think when you see the following math

Perhaps we are talking about the spin of an electron.

$$|\uparrow\rangle \qquad |\downarrow\rangle$$

It can be up or down.

$*$

What to think when you see the following math

We can write a quantum state in terms of some **basis**

$$|\phi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$$

The up and down states form a complete orthogonal basis for an electron spin state.

What to think when you see the following math

These are called **amplitudes**

$$|\phi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$$

They are complex numbers.

$$|\phi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$$

Think of each state as an **outcome** of a measurement

*

What to think when you see the following math

We get the **probability** of the outcome if we square the amplitude.

$$|\alpha|^2 + |\beta|^2 = 1$$

So all of the probabilities must sum to 1.

$*$

Here is what I want you to think when you see this equation:

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$

"ϕ is a state such that, if measured (in this basis) there is a 50% chance of measuring spin up and 50% chance of measuring spin down."

*

Two-particle states

Given two particles with states $|\phi\rangle$ and $|\psi\rangle$, their total state is

$$|\phi\rangle \otimes |\psi\rangle \equiv |\phi\rangle\, |\psi\rangle$$

We will omit the $\otimes$.

$*$

Some clarification on our discussion

While a quantum state can refer to all types of physical situations (electron spin, photon polarization) and can have a basis with more than two vectors, we will focus mainly on states with 2 basis states. Spin-$\frac{1}{2}$ particles are the common example (electrons, neutrons, other elementary fermions) and what we will use.

$*$

Entanglement

"...not one but rather the characteristic trait of quantum mechanics"

– Erwin Schrödinger, on entanglement

A brief history of entanglement

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

– 1935 - A. Einstein, B. Podolsky, N. Rosen

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Entangled singlet pair

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Entangled singlet pair

"EPR paradox"

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What is entanglement mathematically?

Let's start by looking at the following two-particle state:

$$|\Psi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle |\downarrow_2\rangle + |\downarrow_1\rangle |\uparrow_2\rangle \right)$$

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Subscript means between particles 1 and 2

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$$|\Psi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}(\,|\uparrow_1\rangle\,|\downarrow_2\rangle + |\downarrow_1\rangle\,|\uparrow_2\rangle)$$

$$\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} = 50\% \text{ probability (distributed to both states)}$$

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$$|\Psi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}\left(|{\uparrow_1}\rangle |{\downarrow_2}\rangle + |{\downarrow_1}\rangle |{\uparrow_2}\rangle \right)$$

The two possible outcomes of a measurement of this system.

$*$

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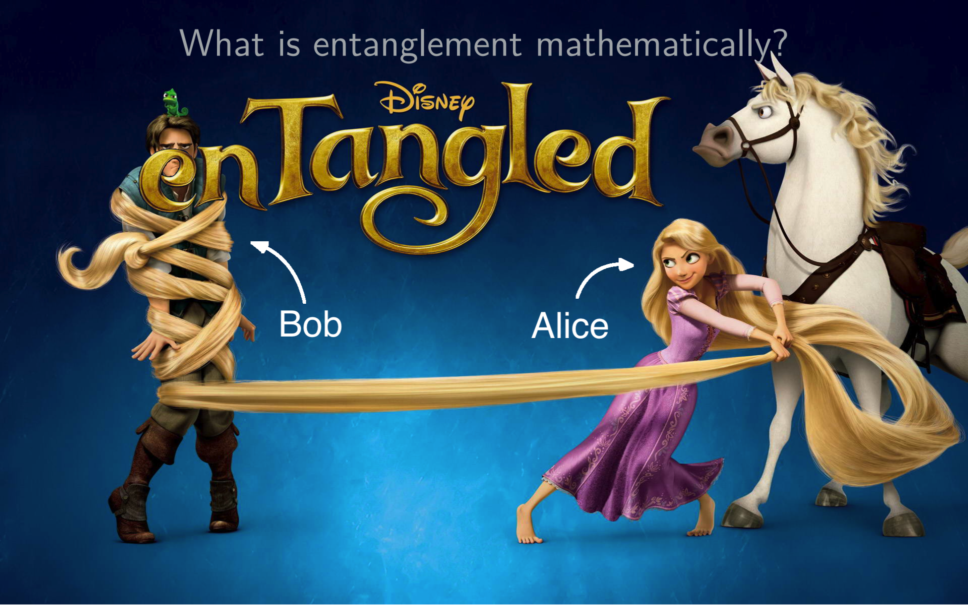

Meet Alice and Bob

What is entanglement mathematically?

$$|\Psi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle \, |\downarrow_2\rangle + |\downarrow_1\rangle \, |\uparrow_2\rangle \right)$$

Alice has particle 1, Bob has particle 2.

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Alice has particle 1, Bob has particle 2.

Let's suppose that Alice makes a measurement of her particle

What is entanglement mathematically?

$$|\Psi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle\, |\downarrow_2\rangle + |\downarrow_1\rangle\, |\uparrow_2\rangle \right)$$

Her result chooses an outcome.

$$*$$

What is entanglement mathematically?

The state we just looked at is a **Bell state**. The Bell states are:

$$|\Psi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle |\downarrow_2\rangle + |\downarrow_1\rangle |\uparrow_2\rangle)$$
$$|\Psi_{12}^{-}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle |\downarrow_2\rangle - |\downarrow_1\rangle |\uparrow_2\rangle)$$
$$|\Phi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle |\uparrow_2\rangle + |\downarrow_1\rangle |\downarrow_2\rangle)$$
$$|\Phi_{12}^{-}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle |\uparrow_2\rangle - |\downarrow_1\rangle |\downarrow_2\rangle)$$

Bell states are the maximally entangled states for two spin-$\frac{1}{2}$ particles.

$*$

Using entanglement for quantum teleportation

"...our teleportation, unlike some science fiction versions, defies no physical laws"

– Bennett et al.

Wait a second, what does teleportation really mean?

Let's start with some context. Our goal is to somehow give someone else an unknown quantum state that we have.

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The no-go theorems:

- No-cloning theorem (no-broadcast theorem)
- No-teleportation theorem
- Physical-transfer-of-a-quantum-state-is-prone-to-corruption-and-attenuation theorem

$*$

Wait a second, what does teleportation really mean?

Quantum teleportation:

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Quantum teleportation:

Like a fax machine

Quantum teleportation:

Like a fax machine where you shred the document immediately after faxing it.

*

The setup

Let's start with the basics - first, we invite Alice and Bob back into the picture.

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We give particle 2 to Alice and particle 3 to Bob.

*

The goal

Give Bob the state $|\phi\rangle$, so that his particle 3 is in the state $|\phi_3\rangle$.

$*$

Going through the steps

So we now have a system of three particles

Alice	**Bob**
particle 1 particle 2 (entangled with 3)	particle 3 (entangled with 2)

Going through the steps

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The state of this system is

$$|\phi_1\rangle\,|\Psi^-_{23}\rangle$$

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The state of this system is

$$\left|\phi_1\right\rangle \left|\Psi^-_{23}\right\rangle$$

Note that the state of particle 1 is separable from the entangled state of particles 2 and 3.

*

Going through the steps

For definiteness let's write our unknown state $|\phi_1\rangle$ in the standard basis:

$$|\phi_1\rangle = \alpha\,|\uparrow_1\rangle + \beta\,|\downarrow_1\rangle$$

where $|\alpha|^2 + |\beta|^2 = 1$.

Going through the steps

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$$|\phi_1\rangle = \alpha\,|{\uparrow_1}\rangle + \beta\,|{\downarrow_1}\rangle$$

where $|\alpha|^2 + |\beta|^2 = 1$.
Using this we can rewrite the total state as

$$\begin{aligned}
|\phi_1\rangle\,|\Psi_{23}^-\rangle &= (\alpha\,|{\uparrow_1}\rangle + \beta\,|{\downarrow_1}\rangle)\frac{1}{\sqrt{2}}(|{\uparrow_2}\rangle\,|{\downarrow_3}\rangle - |{\downarrow_2}\rangle\,|{\uparrow_3}\rangle) \\
&= \frac{\alpha}{\sqrt{2}}(|{\uparrow_1}\rangle\,|{\uparrow_2}\rangle\,|{\downarrow_3}\rangle - |{\uparrow_1}\rangle\,|{\downarrow_2}\rangle\,|{\uparrow_3}\rangle) \\
&+ \frac{\beta}{\sqrt{2}}(|{\downarrow_1}\rangle\,|{\uparrow_2}\rangle\,|{\downarrow_3}\rangle - |{\downarrow_1}\rangle\,|{\downarrow_2}\rangle\,|{\uparrow_3}\rangle)
\end{aligned}$$

$*$

Going through the steps

Ok so now what?

Ok so now what?
We want to get particle 1 involved

$$*$$

Time for a change of basis

$$\frac{\alpha}{\sqrt{2}}(|{\uparrow_1}\rangle\,|{\uparrow_2}\rangle\,|{\downarrow_3}\rangle - |{\uparrow_1}\rangle\,|{\downarrow_2}\rangle\,|{\uparrow_3}\rangle) + \frac{\beta}{\sqrt{2}}(|{\downarrow_1}\rangle\,|{\uparrow_2}\rangle\,|{\downarrow_3}\rangle - |{\downarrow_1}\rangle\,|{\downarrow_2}\rangle\,|{\uparrow_3}\rangle)$$

$$*$$

Time for a change of basis

$$\frac{\alpha}{\sqrt{2}}(\underbrace{|\uparrow_1\rangle\,|\uparrow_2\rangle}\,|\downarrow_3\rangle - \underbrace{|\uparrow_1\rangle\,|\downarrow_2\rangle}\,|\uparrow_3\rangle) + \frac{\beta}{\sqrt{2}}(\underbrace{|\downarrow_1\rangle\,|\uparrow_2\rangle}\,|\downarrow_3\rangle - \underbrace{|\downarrow_1\rangle\,|\downarrow_2\rangle}\,|\uparrow_3\rangle)$$

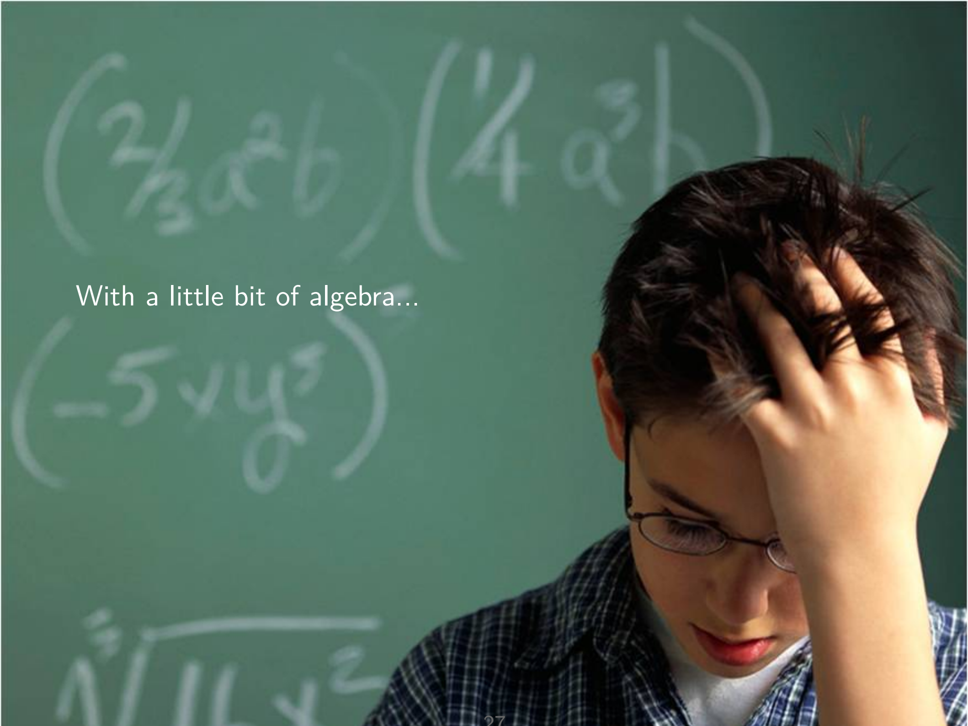
We can write these in terms of the Bell states!

Time for a change of basis

$$\frac{\alpha}{\sqrt{2}}(\underbrace{|\uparrow_1\rangle|\uparrow_2\rangle}_{\frac{|\Phi^+_{12}\rangle+|\Phi^-_{12}\rangle}{\sqrt{2}}}|\downarrow_3\rangle-\underbrace{|\uparrow_1\rangle|\downarrow_2\rangle}_{\frac{|\Psi^+_{12}\rangle+|\Psi^-_{12}\rangle}{\sqrt{2}}}|\uparrow_3\rangle)+\frac{\beta}{\sqrt{2}}(\underbrace{|\downarrow_1\rangle|\uparrow_2\rangle}_{\frac{|\Psi^+_{12}\rangle-|\Psi^-_{12}\rangle}{\sqrt{2}}}|\downarrow_3\rangle-\underbrace{|\downarrow_1\rangle|\downarrow_2\rangle}_{\frac{|\Phi^+_{12}\rangle-|\Phi^-_{12}\rangle}{\sqrt{2}}}|\uparrow_3\rangle)$$

Like so

$*$



With a little bit of algebra...

Going through the steps

$$\frac{1}{2}\Big[\,|\Psi_{12}^{-}\rangle\left(-\alpha\,|{\uparrow_3}\rangle-\beta\,|{\downarrow_3}\rangle\right)$$
$$+\,|\Psi_{12}^{+}\rangle\left(-\alpha\,|{\uparrow_3}\rangle+\beta\,|{\downarrow_3}\rangle\right)$$
$$+\,|\Phi_{12}^{-}\rangle\left(\beta\,|{\uparrow_3}\rangle+\alpha\,|{\downarrow_3}\rangle\right)$$
$$+\,|\Phi_{12}^{+}\rangle\left(-\beta\,|{\uparrow_3}\rangle+\alpha\,|{\downarrow_3}\rangle\right)\Big]$$

$$*$$

Too much!

Let's look at one of the terms:

$$\frac{1}{\sqrt{4}} \left|\Psi^{-}_{12}\right\rangle \left(-\alpha \left|\uparrow_3\right\rangle - \beta \left|\downarrow_3\right\rangle\right)$$

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$$\frac{1}{\sqrt{4}} \left|\Psi_{12}^{-}\right\rangle \left(-\alpha \left|\uparrow_3\right\rangle - \beta \left|\downarrow_3\right\rangle\right)$$

This term has a 1/4 chance of being the outcome of a **measurement** in the Bell basis on particles 1 and 2

Too much!

Let's look at one of the terms:

$$\frac{1}{\sqrt{4}} \left|\Psi_{12}^{-}\right\rangle \left(-\alpha \left|\uparrow_3\right\rangle - \beta \left|\downarrow_3\right\rangle\right)$$

This is a Bell state! So in this outcome, particles 1 and 2 end up entangled.

Too much!

Let's look at one of the terms:

$$\frac{1}{\sqrt{4}} |\Psi^{-}_{12}\rangle \left(-\alpha |{\uparrow_3}\rangle - \beta |{\downarrow_3}\rangle\right)$$

Hey, what does this remind us of?

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Hey, what does this remind us of?

Remember, the desired outcome is $\left|\phi_3\right\rangle = \alpha \left|\uparrow_3\right\rangle + \beta \left|\downarrow_3\right\rangle$

$$*$$

Going through the steps

If we look at this monster again, we notice that all the terms look similar to the one we just looked at!

$$\frac{1}{2}\Big[|\Psi_{12}^{-}\rangle \left(-\alpha\,|{\uparrow_3}\rangle - \beta\,|{\downarrow_3}\rangle\right) \\ + |\Psi_{12}^{+}\rangle \left(-\alpha\,|{\uparrow_3}\rangle + \beta\,|{\downarrow_3}\rangle\right) \\ + |\Phi_{12}^{-}\rangle \left(\beta\,|{\uparrow_3}\rangle + \alpha\,|{\downarrow_3}\rangle\right) \\ + |\Phi_{12}^{+}\rangle \left(-\beta\,|{\uparrow_3}\rangle + \alpha\,|{\downarrow_3}\rangle\right)\Big]$$

$$*$$

Going through the steps

$$\frac{1}{2}\Big[|\Psi^-_{12}\rangle \left(-\alpha\,|{\uparrow_3}\rangle - \beta\,|{\downarrow_3}\rangle\right)$$
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$$+ |\Phi^-_{12}\rangle \left(\beta\,|{\uparrow_3}\rangle + \alpha\,|{\downarrow_3}\rangle\right)$$
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So in this Bell state basis, if we make a measurement on particles 1 and 2, we have four equally-likely outcomes.

Going through the steps

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Going through the steps

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So in this Bell state basis, if we make a measurement on particles 1 and 2, we have four equally-likely outcomes.
In each outcome, particles 1 and 2 end up in a Bell state (entangled), and particle 3 is placed into a pure state very similar to $|\phi_3\rangle$.

Going through the steps

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So in this Bell state basis, if we make a measurement on particles 1 and 2, we have four equally-likely outcomes.
In each outcome, particles 1 and 2 end up in a Bell state (entangled), and particle 3 is placed into a pure state very similar to $\left|\phi_3\right\rangle$.

SO CLOSE!!!

Going through the steps

Here is the state of particle 3 in each outcome:

Outcome 1	Outcome 2	Outcome 3	Outcome 4
$-\alpha \lvert\uparrow_3\rangle - \beta \lvert\downarrow_3\rangle$	$-\alpha \lvert\uparrow_3\rangle + \beta \lvert\downarrow_3\rangle$	$\beta \lvert\uparrow_3\rangle + \alpha \lvert\downarrow_3\rangle$	$-\beta \lvert\uparrow_3\rangle + \alpha \lvert\downarrow_3\rangle$

Going through the steps

If we use the basis $\left\{ \begin{pmatrix} |\uparrow_3\rangle \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ |\downarrow_3\rangle \end{pmatrix} \right\}$ we can write $|\phi_3\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

Outcome 1	**Outcome 2**	**Outcome 3**	**Outcome 4**
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So then we can write these states as:

Outcome 1	**Outcome 2**	**Outcome 3**	**Outcome 4**
$-\alpha\,|{\uparrow_3}\rangle - \beta\,|{\downarrow_3}\rangle$	$-\alpha\,|{\uparrow_3}\rangle + \beta\,|{\downarrow_3}\rangle$	$\beta\,|{\uparrow_3}\rangle + \alpha\,|{\downarrow_3}\rangle$	$-\beta\,|{\uparrow_3}\rangle + \alpha\,|{\downarrow_3}\rangle$

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$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\begin{pmatrix} -\alpha \\ -\beta \end{pmatrix}$	$\begin{pmatrix} -\alpha \\ \beta \end{pmatrix}$	$\begin{pmatrix} \beta \\ \alpha \end{pmatrix}$	$\begin{pmatrix} -\beta \\ \alpha \end{pmatrix}$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$-\, |\phi_3\rangle$	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} |\phi_3\rangle$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} |\phi_3\rangle$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} |\phi_3\rangle$

$*$

Going through the steps

Let's look at the state of particle 3 in the first outcome. It is

$$-|\phi_3\rangle$$

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Any state in the form $e^{i\theta}\left|\phi\right\rangle$ is for our intents equivalent to $\left|\phi\right\rangle$ as $e^{i\theta}$ is just some overall phase. It does not affect the behaviour of the outcomes relative to each other.

$$-\left|\phi_3\right\rangle = e^{i\pi}\left|\phi_3\right\rangle = \left|\phi_3\right\rangle$$

So in the first outcome, Bob's particle is in the state $\left|\phi_3\right\rangle$ already!!! We can teleport a quantum state... well, $1/4$ of the time.

$$*$$

Going through the steps

Looking at the other three 3 outcomes, we notice that each matrix, when squared, is either the identity matrix $\mathbb{I}$ or $-\mathbb{I}$.

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So for example, for the fourth outcome:

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} |\phi_3\rangle = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} |\phi_3\rangle = - |\phi_3\rangle$$

Once again we can ignore the negative sign.

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So we can teleport! Alice just needs to notify Bob of her result.

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TL; DR

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2. Alice takes particle 2, Bob takes particle 3.
3. Alice makes a measurement on her two particles (1 and 2) in the Bell operator basis. There are four equally-likely outcomes.
4. Alice encodes her result in 2 bits and sends it to Bob (directly or via broadcast).

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3. Alice makes a measurement on her two particles (1 and 2) in the Bell operator basis. There are four equally-likely outcomes.
4. Alice encodes her result in 2 bits and sends it to Bob (directly or via broadcast).
5. Based on Alice's result, Bob applies a quantum gate of some sort to his particle 3, or simply does nothing.

TL; DR

Objective Alice wants to send the quantum state $|\phi_1\rangle$, currently on particle 1, to Bob.

Prerequisites Alice and Bob need to share an entangled pair.

1. Start with an EPR singlet of the form $|\Psi^-_{23}\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_2}\rangle\,|{\downarrow_3}\rangle - |{\downarrow_2}\rangle\,|{\uparrow_3}\rangle)$ - an entangled state between particles 2 and 3.
2. Alice takes particle 2, Bob takes particle 3.
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4. Alice encodes her result in 2 bits and sends it to Bob (directly or via broadcast).
5. Based on Alice's result, Bob applies a quantum gate of some sort to his particle 3, or simply does nothing.
6. Bob's particle 3 is now in the state $|\phi_3\rangle$ as desired.

Side effects Particles 1 and 2 are now entangled in a Bell state.

$*$

The comedown

"Beam me up, Scotty"

– Captain Kirk

So where are we at?

So where are we at?
Is teleportation actually possible?

So where are we at?
Is teleportation actually possible?

Yes.

*

Questions?